

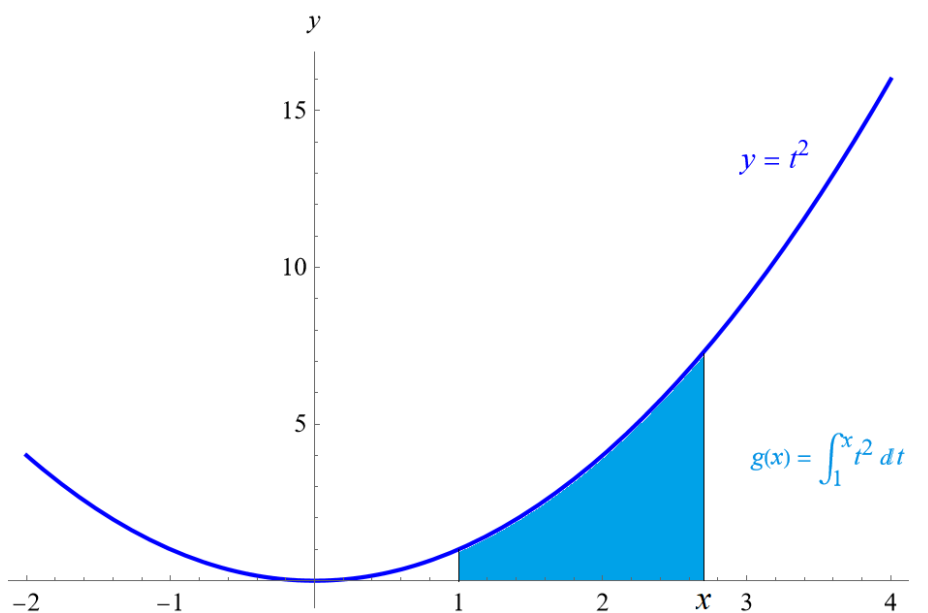
## Exercise 5

Sketch the area represented by  $g(x)$ . Then find  $g'(x)$  in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_1^x t^2 dt$$

### Solution

$g(x)$  is the area under the curve  $y = t^2$  from  $t = 1$  to  $t = x$ .



### Part (a)

Get  $g'(x)$  by taking the derivative of both sides and using part 1 of the fundamental theorem of calculus.

$$\begin{aligned} g'(x) &= \frac{d}{dx} \int_1^x t^2 dt \\ &= x^2 \end{aligned}$$

**Part (b)**

Evaluate the integral first using part 2 of the fundamental theorem of calculus

$$\begin{aligned}g(x) &= \int_1^x t^2 dt \\&= \left. \frac{t^3}{3} \right|_1^x \\&= \frac{x^3}{3} - \frac{1^3}{3} \\&= \frac{x^3}{3} - \frac{1}{3}\end{aligned}$$

and then take the derivative of both sides.

$$\begin{aligned}g'(x) &= \frac{d}{dx} \left( \frac{x^3}{3} - \frac{1}{3} \right) \\&= x^2\end{aligned}$$

The same answer is obtained either way.